Towards NNLO QCD + NLO EW Drell-Yan Predictions with GENEVA

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UNIMIB & INFN

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Outline

Motivation & State of the Art

The GENEVA Method

Extending GENEVA to NLO EW

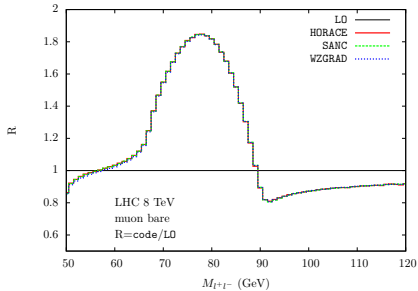
Preliminary phenomenology

Summary & Outlook

Backup

Why Drell-Yan?

- Among the cleanest, highest-statistics processes at the LHC
- Precision Standard Model laboratory:
  - m_W and $\sin^2\theta_W^{\text{eff}}$ extractions
  - PDF determination, luminosity monitor
  - EFT/new-physics searches in the high-mass tail
- Percent-level experimental precision $\Rightarrow$ theory must match: both **NNLO QCD** *and* **NLO EW** corrections are needed simultaneously
- A fully-differential, matched-to-parton-shower prediction is more versatile and what experiments use



NLO EW/LO, $M_{\mu^+\mu^-}$ near the Z peak

[1606.02330]

FSR shifts leptons to lower energy $\Rightarrow$ depletes the resonance peak, piles events up below it

QCD precision frontier for Drell-Yan

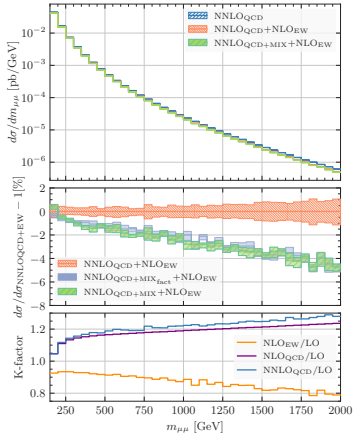
- Fixed-order QCD now pushed to $\mathrm{N^3LO}$ for total/rapidity-differential DY
- Multiple independent codes cross-validated at NNLO/$\mathrm{N^3LO}$, including resolving fiducial-cut/p_T-cut instabilities via resummation [2405.19714]
- But a fixed-order calculation alone is not what experiments compare to:
  - needs resummation of logs at small p_T/small jettiness
  - needs a parton shower for hadron-level predictions
- $\Rightarrow$ NNLO+PS matching is now the standard precision target

Landscape of NNLO+PS matching methods

- Three main approaches achieve NNLO QCD matched to a parton shower:
  MiNNLO$_{\mathrm{PS}}$: MiNLO-improved multi-scale matching
  UN^{2}LOPS: NNLO via unitarity-preserving subtraction
  Geneva: N-jettiness subtraction + SCET resummation + shower
- Initially applied to simple color-singlet processes (Drell-Yan, $gg \to H$), now extended to diboson, $t\bar{t}$, HH, ...
- Experimental analyses often apply bin-by-bin NLO EW K-factors to QCD events
- $\implies$ better to include EW corrections directly in the event generator, retaining full phase-space dependence.

EW precision frontier for Drell-Yan

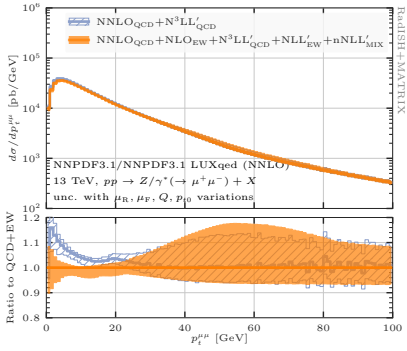
- NLO EW corrections to DY known for over a decade [0911.2329] [1103.0914]
- Two-loop **mixed QCD×EW** fixed-order corrections now complete:
  - neutral current [2106.11953] [2201.01754] [2412.16095]
  - charged current [2405.00612]
- Concrete size (relative to NLO QCD), neutral-current DY:
  - $\sim -15\%$ at $p_T^\ell \sim 500$ GeV
  - $\sim -1.5\%$ up to $m_{\ell\ell} \sim 1$ TeV
- EW Sudakov logs grow like $\alpha_e \ln^2(Q^2/m_W^2)$ large at the TeV scale [1511.08692]



NNLO QCD + mixed QCD×EW + NLO EW, high-mass $m_{\mu\mu}$ [2412.16095]

Stress test: EW corrections vs. NNLO QCD

- NNLO QCD scale uncertainties on fiducial DY observables: typically $\lesssim$ 1–2% away from extreme phase-space corners
- EW correction sizes are **comparable to or larger** than these:
  - $p_{T,\ell\ell} \lesssim 10$ GeV: EW resummed effects shift the spectrum by 10–15% relative to the N^{3}LL′+NNLO QCD baseline [2404.15112]
  - high $p_T^\ell/m_{\ell\ell}$: mixed QCD×EW corrections reach -15% ($p_T^\ell \sim 500$ GeV) [2106.11953]



[2404.15112]

⇒ EW corrections are **not** a sub-percent afterthought: precision DY phenomenology *requires* NNLO QCD and NLO EW **together**, consistently matched to a shower

How to combine QCD and EW

- **Additive** (total cross section): $\sigma = \sigma_{\text{NLO QCD}} + \sigma_{\text{NLO EW}} - \sigma_{\text{LO}}$
  - POWHEG `Z_ew-BMNNPV` [1302.4606] [2402.14659]: radiation itself is *not* additive as QCD/EW emissions compete via their Sudakov form factors
  - RADISH RES+FO [2404.15112]
- **Approximate factorized**: EW as a multiplicative K-factor on top of QCD
  - MEPS@NLO EW-virtual approximation [1511.08692]
- **Merged / interleaved showers**: QCD+EW showered together
  - resonance-aware MC@NLO QCD+EW in SHERPA [2607.06059]
  - "towards" QED in MINLO [2602.16029]
- **Joint SCET factorization** (**this talk**): QCD *and* EW channels summed inside the *same* hard×beam×beam×soft factorization using an appropriate resolution variable → GENEVA's approach

GENEVA in a nutshell

GENEVA consistently combines three ingredients [1311.0286] [1508.01475]:

1. fully-differential **NNLO QCD** via a resolution parameter
   - N-jettiness $\mathcal{T}_N$ [1311.0286]
   - transverse momentum q_T [2102.08390]
   - jet veto p_T^{jet} [2308.11577]
   - ...
2. higher-order **resummation** (NNLL′) from Soft-Collinear Effective Theory
3. matching to a **parton shower** (PYTHIA8/Vincia), preserving both accuracies

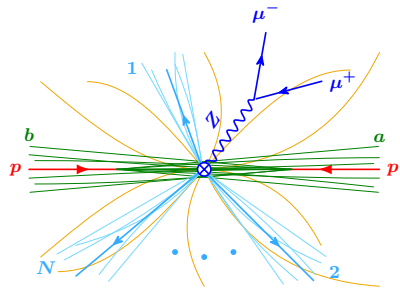
$$\Rightarrow \text{NNLO+NNLL}'\text{+PS event generator}$$

N-jettiness definition

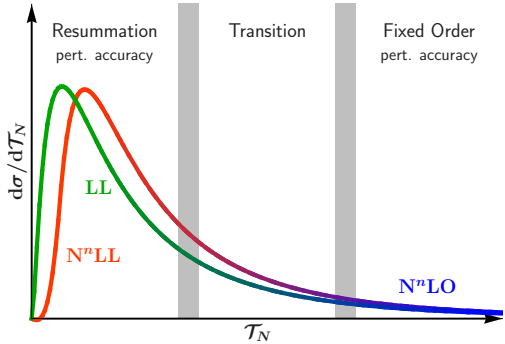
- **global event shape** quantifying how close a configuration is to exactly N jets
- vanishing in the N-jet limit, growing with extra radiation

$$\mathcal{T}_N = \min_{\hat{q}_1,\ldots,\hat{q}_N} \left\{ \sum_{k\in(\text{quark, gluon})} \min\{\hat{q}_a \cdot p_k,\ \hat{q}_b \cdot p_k,\ \hat{q}_1 \cdot p_k,\ \ldots,\ \hat{q}_N \cdot p_k\} \right\},\ \hat{q}_i \text{ light-like axes}$$

- $\mathcal{T}_N \to 0$ captures the QCD IR-singular limit
- **ordering**: $\mathcal{T}_{N+1} < \mathcal{T}_N$, so an IR divergence in $\mathcal{T}_N$ is a divergence in $\mathcal{T}_{N+1}$ too
- clusters all particles into N-jets and beam regions in an IR-safe way $\implies$ no need for an additional jet algo to find jet axes [1508.01516]
- well studied with known factorization, singular structure and resummation [0709.2709] [0711.2079] [0910.0467] [1004.2489]



$\mathcal{T}_0$ spectrum

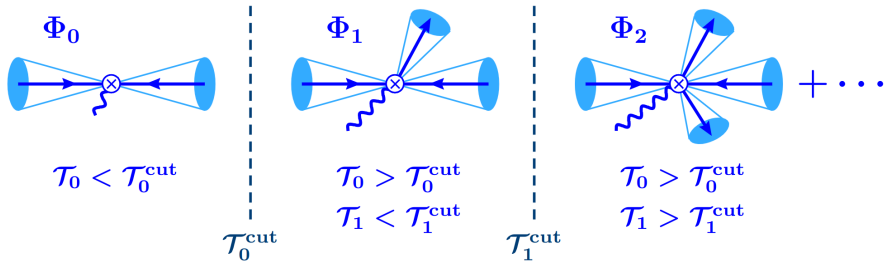


- **peak region:** resummation effects are most important and nonsingular is power suppressed
- **transition region:** interplay between resummed and nonsingular, i.e. both are equally important
- **tail region:** resummation makes no sense here and must be turned off, spectrum predicted by the FO

[1311.0286]

Event definition: N-jettiness partition

Partition the phase space Φ into N-jet bins Φ_N, take $\mathcal{T}_N^{\text{cut}} \to 0$, and assign to each event a Monte Carlo cross section

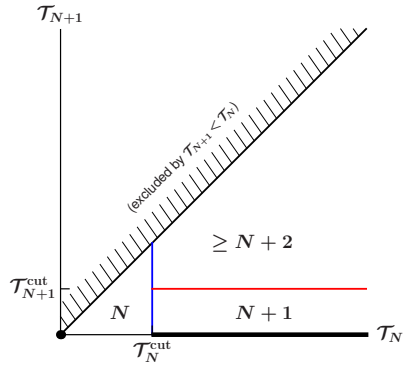


$$\mathrm{d}\sigma = \mathrm{d}\Phi_0 \frac{\mathrm{d}\sigma_0^{\text{MC}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \mathrm{d}\Phi_1 \frac{\mathrm{d}\sigma_1^{\text{MC}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}; \mathcal{T}_1^{\text{cut}}) + \mathrm{d}\Phi_2 \frac{\mathrm{d}\sigma_{\geq 2}^{\text{MC}}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}; \mathcal{T}_1 > \mathcal{T}_1^{\text{cut}})$$

- splitting phase space into exclusive N-, $(N{+}1)$-jet and inclusive $(N{+}2)$-jet bins
- Emissions with $\mathcal{T}_N(\Phi_M) < \mathcal{T}_N^{\text{cut}}$ are unresolved and parametrized by $\Phi_N(\Phi_M)$
- Emissions with $\mathcal{T}_N(\Phi_M) > \mathcal{T}_N^{\text{cut}}$ are resolved and parametrized by the exact Φ_M

Event definition: N-jettiness partition

- As $\mathcal{T}_N^{\text{cut}}, \mathcal{T}_N \to 0$, the corresponding logs deteriorate the perturbative convergence $\Rightarrow$ resummation
- Higher-order resummation will improve the accuracy of the predictions across the whole spectrum
- $\mathcal{T}_N^{\text{cut}}$ is *physical*: it becomes the parton-shower starting scale, not an unphysical slicing parameter sent to zero



[1311.0286]

SCET QCD Factorization for $\mathcal{T}_0$

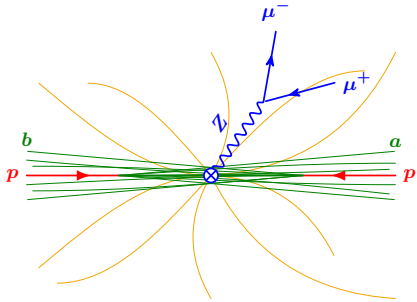
in terms of hard, collinear and soft contributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_0} = H_{ij}(\mu) \int \mathrm{d}t_a \mathrm{d}t_b \, B_i(t_a, \mu) B_j(t_b, \mu) S(\mathcal{T}_0 - t_a/Q - t_b/Q, \mu) + \ldots$$

Hard func' H_{ij}: process dependent, captures virtual corrections at $\mu_H \sim Q$

Beam funcs' B_i: encode collinear ISR radiation at $\mu_B \sim \sqrt{Q\mathcal{T}_0}$ and contain the PDFs

Soft func' S: captures wide-angle soft radiation at $\mu_S \sim \mathcal{T}_0$ and depends only on the color representation of the hard partons



Resummed $\mathcal{T}_0$ spectrum

To perform $\mathcal{T}_0$ resummation:

1. evaluate H, S, B at their canonical scales μ_H, μ_S, μ_B such that their log structure vanishes
2. solve the renormalization group equations

$$\mu \frac{\mathrm{d}X}{\mathrm{d}\mu} = \gamma_X \otimes X\,, \qquad X = H, S, B$$

and perform the evolution from $\mu_{H,S,B}$ to any μ

We thus obtain the resummed $\mathcal{T}_0$ spectrum:

$$\frac{\mathrm{d}\sigma^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} = \sum_{ij} \frac{\mathrm{d}\sigma^B_{ij}}{\mathrm{d}\Phi_0} H_{ij}(Q^2, \mu_H)\, U_H(\mu_H, \mu) \otimes \left[S(\mu_S)\, U_S(\mu_S, \mu)\right] \otimes \left[B_i(x_a, \mu_B)\, U_B(\mu_B, \mu)\right] \otimes \left[B_j(x_b, \mu_B)\, U_B(\mu_B, \mu)\right]$$

[1909.02026]

Matching of resummed and fixed-order results

Schematically, the Monte Carlo cross sections are given by

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0} - \frac{\mathrm{d}\sigma_0^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0}\Big|_{\mathcal{O}(\alpha_s^2)}\right](\mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_0^{\mathrm{NNLO}}}{\mathrm{d}\Phi_0}$$

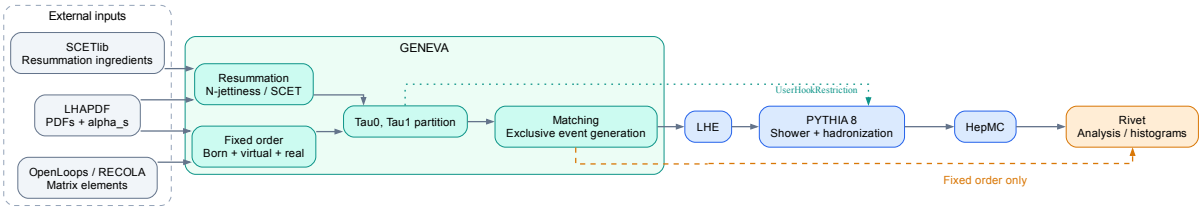
$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} - \frac{\mathrm{d}\sigma_0^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\Big|_{\mathcal{O}(\alpha_s^2)}\right]\theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\mathcal{P}(\Phi_1) + \frac{\mathrm{d}\sigma_1^{\mathrm{NLO}}}{\mathrm{d}\Phi_1}$$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} - \frac{\mathrm{d}\sigma_0^{\mathrm{NNLL'}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\Big|_{\mathcal{O}(\alpha_s^2)}\right]\theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})\mathcal{P}(\tilde{\Phi}_1)\mathcal{P}(\Phi_2) + \frac{\mathrm{d}\sigma_2^{\mathrm{LO}}}{\mathrm{d}\Phi_2}$$

- NNLL′ resummed − its own fixed-order expansion = pure resummation content, free of double counting
- + full fixed order: restores the exact FO normalization
- $\mathcal{P}(\Phi_1)$: normalized splitting probability extending the 0-jet resummation to the 1-, 2-jet samples

[2010.10498]

GENEVA pipeline



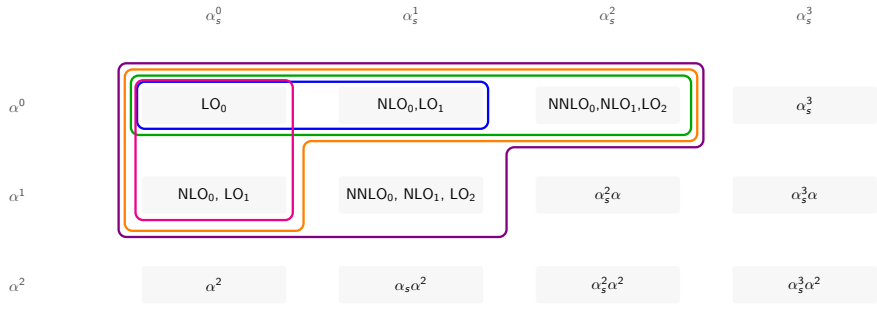
- Constrained parton shower preserves the perturbative accuracy of the matched calculation
- Fixed-order or showered histograms
- Produces LHE or HepMC events

Why GENEVA for QCD+EW?

- "The formalism can be naturally extended ... to include electroweak corrections", GENEVA methods paper, 2025 [2504.11357]
- GENEVA's *N*-jettiness factorization is a generic SCET statement: it does not care whether the radiation is a gluon or a photon
- Opportunity: build a *joint* QCD+EW factorization, rather than bolting NLO EW onto an NNLO QCD result after the fact
- Work in progress on a dedicated QCD+EW development line in GENEVA: this talk reports on where that stands

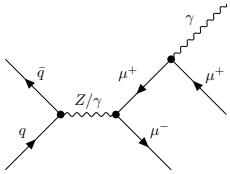
Order counting in QCD+EW

Drell-Yan, factoring out the Born contribution α^2:

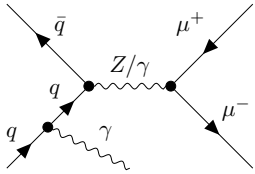


- **Democratic counting**: the subscript j counts resolved QCD and QED radiation equally
- by naive coupling counting $\alpha \sim \alpha_S^2$:
  - NLO EW $\sim$ NNLO QCD
  - mixed QCD×EW $\sim$ N^{3}LO QCD

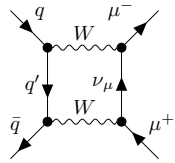
Drell-Yan NLO EW contributions



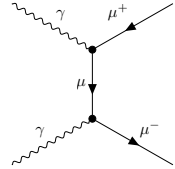
NLO EW real, FSR



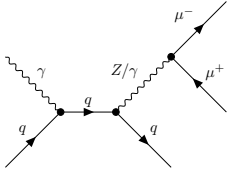
NLO EW real, ISR



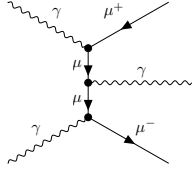
NLO EW virtual



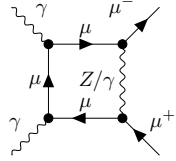
LO, photon-induced



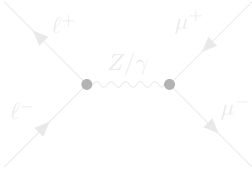
NLO EW real,
photon-induced



NLO EW real,
photon-induced

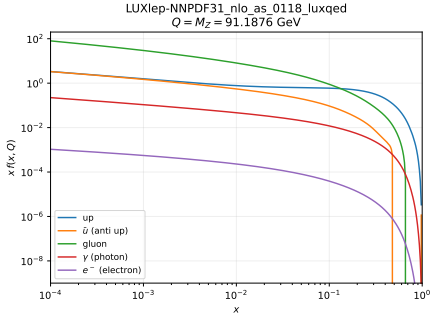


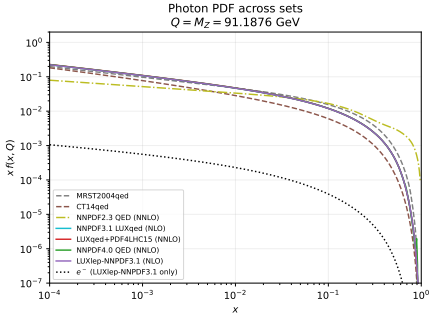
NLO EW virtual,
photon-induced



LO & NLO,
lepton-induced
(neglected)

Photon & lepton PDFs at the Z pole





- Photon PDF: ~ 2 orders of magnitude below the quarks at $Q = M_Z$
- All modern LUXQED-based sets **overlap almost exactly**
- Pre-LUXQED sets disagreed by up to a factor ~ 2 [1607.04266] [1708.01256]
- Electron PDF a further $\sim$2–3 orders down [2005.06477]
  $\Rightarrow$ lepton-induced channels are neglected, but a priori straightforward to extend should experimental precision call for it
- NNPDF4.0QED [2401.08749] applies the same LUXQED formula with a reduced $Q_{\text{min}} = 1.65$ GeV instead of 10 GeV

Including NLO+NLL EW in GENEVA

Schematically,

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{\mathrm{NNLL',NLL}}}{\mathrm{d}\Phi_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL',NLL}}}{\mathrm{d}\Phi_0}\Big|_{\mathcal{O}(\alpha_s^2),\mathcal{O}(\alpha)} + \frac{\mathrm{d}\sigma^{\mathrm{NNLO_0,NLO_0}}}{\mathrm{d}\Phi_0}$$

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma^{\mathrm{NNLL',NLL}}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL',NLL}}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0}\Big|_{\mathcal{O}(\alpha_s^2),\mathcal{O}(\alpha)}\right]\mathcal{P}(\Phi_1) + \frac{\mathrm{d}\sigma^{\mathrm{NLO_1,LO_1}}}{\mathrm{d}\Phi_1}$$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \left[\frac{\mathrm{d}\sigma^{\mathrm{NNLL',NLL}}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL',NLL}}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0}\Big|_{\mathcal{O}(\alpha_s^2),\mathcal{O}(\alpha)}\right]\mathcal{P}(\tilde{\Phi}_1)\mathcal{P}^{\mathrm{QCD}}(\Phi_2) + \frac{\mathrm{d}\sigma^{\mathrm{LO_2,-}}}{\mathrm{d}\Phi_2}$$

- $\mathrm{d}\sigma^{\text{FO QCD}_i\text{, FO EW}_i}$ additive fixed order in Φ_i
- $\mathrm{d}\sigma^{\text{Resum QCD, Resum EW}}$ and its QCD and EW fixed-order expansion relative to the Born order
- $\mathcal{P}(\Phi_1)$: normalized splitting probability in both QCD and EW, while $\mathcal{P}^{\mathrm{QCD}}(\Phi_2)$ is only QCD

Democratic $\mathcal{T}$: from QCD to QCD+EW

$\mathcal{T}_N$ is extended to include *all* QCD and EW radiation

$$\mathcal{T}_0 = \sum_k \min\{\hat{q}_a \cdot p_k,\ \hat{q}_b \cdot p_k\} \qquad \mathcal{T}_1 = \min_{\hat{q}_1} \left\{ \sum_k \min\{\hat{q}_a \cdot p_k,\ \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k\} \right\}$$

- **Pure QCD**: k runs over radiated gluons and light quarks $\{g, q, \bar{q}\}$
- **QCD+EW**: the sum is extended to *all* QCD *and* QED radiation, $k \in \{g, q, \bar{q}\} \cup \{\gamma\}$
- **Crucial exclusion**: the **massive** DY leptons $\mu^{\pm}$ *do not* enter this sum at any order

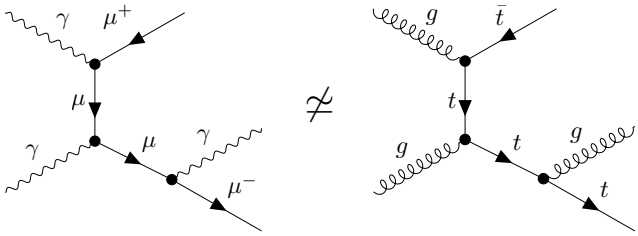
Order	**Radiation in k**	**Status**
NLO QCD (α_s)	+ one g or $q(\bar{q})$	
NNLO QCD (α_s^2)	+ incl. $g \to q\bar{q}$ splitting	
NLO EW (α_e)	+ one γ	**this talk**
NNLO EW (α_e^2)	+ incl. $\gamma \to \ell'\bar{\ell}'$ splitting	not yet

Physical setup: keeping massive leptons

- QED final-state radiation is inclusive over *both* beam and lepton directions $\Rightarrow$ naively forces $\mathcal{T}_0 \to \mathcal{T}_2$
- The $\mathcal{T}_2$ EW soft-function boundary term is known to NLO and NNLO QCD [2312.11626]
- **Our choice**: keep the leptons *massive* ($m_\mu \neq 0$) $\Rightarrow$ $\mathcal{T}_0$ remains a valid resolution variable
- Adapt the massive-particle SCET factorization built for $t\bar{t}$ production [2111.03632]
- Price to pay: unresummed $\ln(m_\mu/Q)$ logs
- Independent precedent: POWHEG's Z_ew_BMNNPV implementations also keep leptons massive, as "the true physical regulator of QED mass singularities" [1202.0465] [1302.4606]

Photon-induced processes: $\gamma\gamma \to \mu^+\mu^-(\gamma)$

- $\gamma\gamma \to \mu^+\mu^-$ is a genuine **LO** channel (via the proton's photon PDF), percent-level contribution to NC DY [1606.02330]
- For $\gamma\gamma \to \mu^+\mu^-\gamma$: no alternative (massless, charged) emitter exists for the radiated photon to be collinear to, unlike $q\bar{q} \to \mu^+\mu^-\gamma$, where it could also be collinear to an initial-state quark



- **Why QCD escapes this**: in $gg \to t\bar{t}\,g$, a gluon collinear to a massive top can equivalently come from an *initial-state* gluon via the *ggg* vertex
- QED has **no** $\gamma\gamma\gamma$ vertex (abelian, photon uncharged)

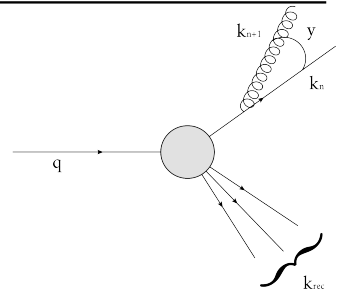
$\Rightarrow$ massive-lepton emitter regions are **mandatory** for $\gamma\gamma \to \mu^+\mu^-\gamma$

Massive-emitter FKS mapping

- Massive FSR emitter $\Rightarrow$ the real$\to$Born phase-space map is not unique
- GENEVA implements two prescriptions:

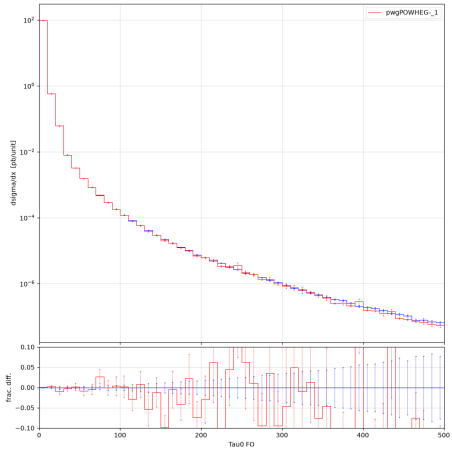
Mapping	**Born emitter axis**	**Angle var.**	**Status**
Buonocore et al. [1711.06281]	$\parallel (k_n + k_{n+1})$	FKS (ξ, y)	**default**, feeds resum.
Barzè et al. [1202.0465]	$\parallel k_n$	linear z	FO cross-check

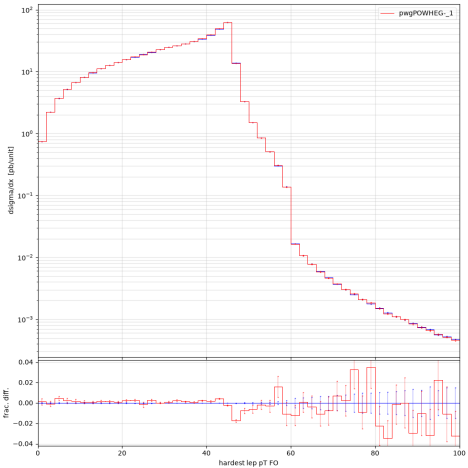
- Both are *exact, bijective* reparametrizations, i.e. recoil conventions
- Apply finite quasi-collinear subtraction to tame large, unresummed mass logarithms in the fixed-order remainder.
- Cross-checked against each other in GENEVA's FKS mapping tests



k_n: massive emitter, k_{n+1}: radiated parton, k_{rec}: recoil system [1711.06281]

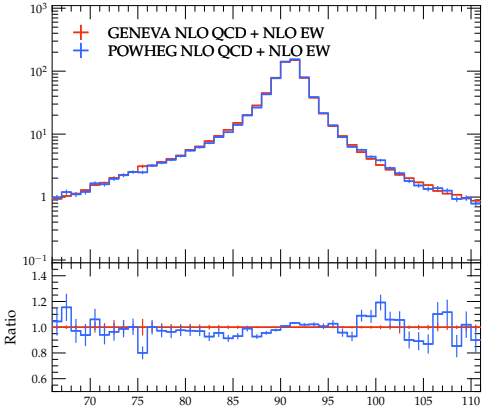
Preliminary: NLO EW validation [1302.4606]

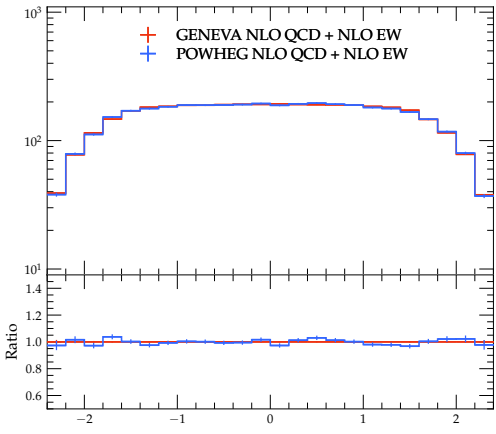




- Per mille agreement with POWHEG NLO EW

Preliminary: NLO QCD + NLO EW validation [1302.4606]





- Combination of NLO QCD + NLO EW in GENEVA agrees with POWHEG

Joint QCD+QED factorization theorem for $\mathcal{T}_0$

$$\frac{d\sigma}{d\Phi_0\, d\mathcal{T}_0} = \sum_{\kappa \in \kappa^{0j}_{\rm QCD},\, \kappa^{0j}_{\rm EW}} H_\kappa(\Phi_0, \mu) \int dt_a\, dt_b\; B_a(t_a, z_a, \mu)\, B_b(t_b, z_b, \mu)$$
$$\times\; S_\kappa(\Phi_0, \mathcal{T}_0 - t_a/Q_A - t_b/Q_B, \mu)\left[1 + \mathcal{O}(\mathcal{T}_0/Q)\right]$$

with

$$\kappa^{0j}_{\rm QCD} = \{q\bar{q},\, \bar{q}q,\, gg\}\,, \qquad \kappa^{0j}_{\rm QED} = \{f\bar{f},\, \bar{f}f,\, \gamma\gamma\}$$

- QCD+QED beam functions include quark, antiquark, and photon PDFs
- Charged final-state leptons introduce additional QED soft Wilson lines
- Leading DY channel: $q\bar{q} \to Z/\gamma^* \to \ell^+\ell^-$
- Photon-induced $\gamma\gamma \to \ell^+\ell^-$ is a separate tree-level Born channel
- Loop-induced $gg \to Z^* \to \ell^+\ell^-$ is suppressed

Coupled α_s–α_e running & abelianization

- Mixed QCD-EW evolution needs *coupled* RGEs for $\alpha_s(\mu)$ and $\alpha_e(\mu)$ [1907.02971]: there is no closed-form evolution kernel, and $\alpha_s(\mu), \alpha_e(\mu)$ are instead iterated numerically and fed into a numerical μ-integration
- Strategy: obtain EW ingredients from the known QCD ones by **abelianization**
- Replaces a color-charge *dot product between a pair of partons* by their electric charges,

$$\boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\longrightarrow\; Q_i\, Q_j\,, \qquad Q_i \to -|Q_i| \text{ for an antiparticle,}$$

- Works cleanly for beam functions (initial-state partons only)
- *Fails* for hard/soft functions: once photons enter, the final-state leptons are themselves charged, giving new eikonal correlations with the outgoing legs that the QCD soft function (blind to the colorless lepton final state) never had
  - instead: abelianize the $t\bar{t}$ massive-particle results [2111.03632]

Resummed $\mathcal{T}_0$ spectrum: QCD+QED

Same resummation strategy as pure QCD, now with *coupled* α_s, α_e running:

$$\frac{\mathrm{d}\sigma^{\mathrm{NNLL}',\mathrm{NLL}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} = \sum_{\kappa \in \kappa^{0j}_{\mathrm{QCD}}, \kappa^{0j}_{\mathrm{EW}}} \frac{\mathrm{d}\sigma^B_\kappa}{\mathrm{d}\Phi_0} H_\kappa(Q^2, \mu_H)\, U_H(\mu_H, \mu) \otimes \left[S_\kappa(\mu_S)\, U_S(\mu_S, \mu)\right]$$
$$\otimes \left[B_a(x_a, \mu_B)\, U_B(\mu_B, \mu)\right] \otimes \left[B_b(x_b, \mu_B)\, U_B(\mu_B, \mu)\right]$$

- Same $\mu\, \mathrm{d}X/\mathrm{d}\mu = \gamma^\kappa_X(\alpha_s, \alpha_e) \otimes X$ structure as pure QCD, but now γ^κ_X depends on *both* couplings, summed over QCD and EW channels κ
- Coupled $\beta_{\alpha_s}, \beta_{\alpha_e} \Rightarrow$ no closed-form μ-integration: the evolution kernels U_X are evaluated semi-numerically

[1909.02026]

The N-jettiness hard function: RGE structure

The Wilson coefficient / hard function obeys a simple **multiplicative** RGE,

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\,C^\kappa(\mu) = \gamma_C^\kappa(\mu)\,C^\kappa(\mu)\,, \qquad \gamma_C^\kappa(\mu) = \Gamma^\kappa(\alpha_s,\alpha_e)\,L + \gamma_C^\kappa(\alpha_s,\alpha_e)$$

with $L = \ln\!\big(Q^2/\mu^2\big)$, and $\gamma_H^\kappa(\mu) = 2\,\mathrm{Re}\big[\gamma_C^\kappa(\mu)\big]$ relating it to the hard function.

- cusp (Γ^κ) and noncusp (γ_C^κ) anomalous dimensions differ case by case, with only their *functional* μ-dependence universal
- solved order-by-order in α_s, α_e using the coupled α_s–α_e RGEs $\Rightarrow$ closed-form, all-orders recursive solution
- the beam/soft functions obey the *same* coupled-RGE logic

[1907.02971][1909.00811]

Recursive solution: the $\mathcal{O}(\alpha_s\alpha_e)$ hard function

Valid to *all orders* in α_s, α_e .

$$C^{\kappa}_{(n,m)}(\mu) = \int_Q^{\mu} \frac{\mathrm{d}\bar{\mu}}{\bar{\mu}} \sum_{(k,\ell)=(0,0)}^{(n,m)} \Big\{ \tilde{\Theta}(n{-}k{-}1)\, 2k\, \beta^{s}_{(n-k,m-\ell)} + \tilde{\Theta}(m{-}\ell{-}1)\, 2\ell\, \beta^{e}_{(m-\ell,n-k)} + \tilde{\Theta}(n{-}k{+}m{-}\ell{-}1) \Big[\Gamma^{\kappa}_{(n-k,m-\ell)} \ln \frac{Q^2}{\bar{\mu}^2} + \gamma^{\kappa}_{C\,(n-k,m-\ell)} \Big] \Big\}\, C^{\kappa}_{(k,\ell)}(\bar{\mu}) + c^{\kappa}_{(n,m)}$$

At $\mathcal{O}(\alpha_s\alpha_e)$ the hard-function coefficient is

$$\begin{aligned} H^{\kappa}_{(1,1)}(\mu) = {} & \tfrac{1}{4} H^{\kappa}_{(0,0)} \Gamma^{\kappa}_{(1,0)} \Gamma^{\kappa}_{(0,1)}\, \tilde{L}^4 + \tfrac{1}{4} H^{\kappa}_{(0,0)} \big(\gamma^{\kappa}_{H(1,0)} \Gamma^{\kappa}_{(0,1)} + \gamma^{\kappa}_{H(0,1)} \Gamma^{\kappa}_{(1,0)} \big) \tilde{L}^3 \\ & + \tfrac{1}{4} \Big[H^{\kappa}_{(0,0)} \big(\gamma^{\kappa}_{H(0,1)} \gamma^{\kappa}_{H(1,0)} - 2\Gamma^{\kappa}_{(1,1)} \big) - 2\Gamma^{\kappa}_{(1,0)} h^{\kappa}_{(0,1)} - 2\Gamma^{\kappa}_{(0,1)} h^{\kappa}_{(1,0)} \Big] \tilde{L}^2 \\ & - \tfrac{1}{2} \big(H^{\kappa}_{(0,0)} \gamma^{\kappa}_{H(1,1)} + \gamma^{\kappa}_{H(1,0)} h^{\kappa}_{(0,1)} + \gamma^{\kappa}_{H(0,1)} h^{\kappa}_{(1,0)} \big) \tilde{L} + h^{\kappa}_{(1,1)} \end{aligned}$$

- $\Gamma^{\kappa}_{(1,1)}$, $\gamma^{\kappa}_{H(1,1)}$, and terms like $\Gamma^{\kappa}_{(1,0)} h^{\kappa}_{(0,1)}$ are *genuinely mixed* ingredients, absent from both the pure-QCD and pure-EW results

[1907.02971][1909.00811]

Validating the singular/nonsingular

- The nonsingular piece is defined as the *full* fixed-order result minus the resummed result expanded to that same fixed order:

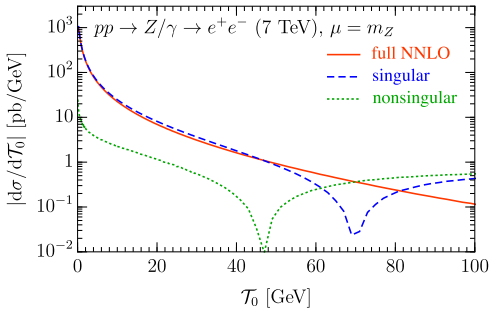
$$\underbrace{\frac{d\sigma_0^{\text{nons}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}})}_{\text{nonsingular}} = \underbrace{\frac{d\sigma_0^{\text{NNLO}_0}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}})}_{\text{full FO}} - \underbrace{\frac{d\sigma_0^{\text{NNLL}'}}{d\Phi_0}\Big|_{\mathcal{O}(\alpha_s^2)}(\mathcal{T}_0^{\text{cut}})}_{\text{singular}}$$

- Calibrated against an *independent* total NNLO cross section: the exclusive 0-jet rate below $\mathcal{T}_0^{\text{cut}}$ is reweighted so that $\sigma_0(\mathcal{T}_0^{\text{cut}}) + \sigma_{\geq 1}(\mathcal{T}_0^{\text{cut}})$ reproduces the external NNLO total
- Cross-check: the nonsingular cumulant $\sigma_{\text{nons}}(\mathcal{T}_0^{\text{cut}})$ must vanish as $\mathcal{T}_0^{\text{cut}} \to 0$, order by order in α_s
- At the **spectrum** level (what the following validation plots check), the same split reads

$$\underbrace{\frac{d\sigma_1^{\text{nons}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})}_{\text{nonsingular}} = \underbrace{\frac{d\sigma_1^{\text{NLO}_1,\text{LO}_1}}{d\Phi_1}(\mathcal{T}_0)}_{\text{full FO}} - \underbrace{\frac{d\sigma_0^{\text{NNLL}',\text{NLL}}}{d\Phi_0\, d\mathcal{T}_0}\Big|_{\mathcal{O}(\alpha_s^2),\mathcal{O}(\alpha)} \mathcal{P}(\Phi_1)\,\theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})}_{\text{singular}}$$

[1508.01475]

QCD Resummation and profiles



[1508.01475]

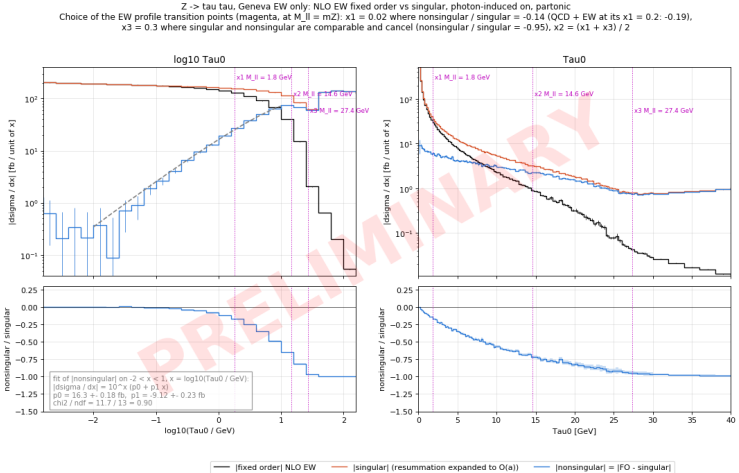
$$\mu_S(\mathcal{T}_0) = \mu_{\rm FO}\, f_{\rm run}(\mathcal{T}_0/Q)\,,$$
$$\mu_B(\mathcal{T}_0) = \mu_{\rm FO}\, \sqrt{f_{\rm run}(\mathcal{T}_0/Q)}\,,$$
$$\mu_H = \mu_{\rm FO}$$

$$f_{\rm run}(x) = \begin{cases} x_0\left[1 + (x/x_0)^2/4\right] & x \le 2x_0 \\ x & 2x_0 \le x \le x_1 \\ x + \dfrac{(2-x_2-x_3)(x-x_1)^2}{2(x_2-x_1)(x_3-x_1)} & x_1 \le x \le x_2 \\ 1 - \dfrac{(2-x_1-x_2)(x-x_3)^2}{2(x_3-x_1)(x_3-x_2)} & x_2 \le x \le x_3 \\ 1 & x_3 \le x \end{cases}$$

$$\mu_{\rm FO} = Q,\ x_0 = 2.5\ \mathrm{GeV}/Q,\ \{x_1, x_2, x_3\} = \{0.2, 0.5, 0.8\}$$

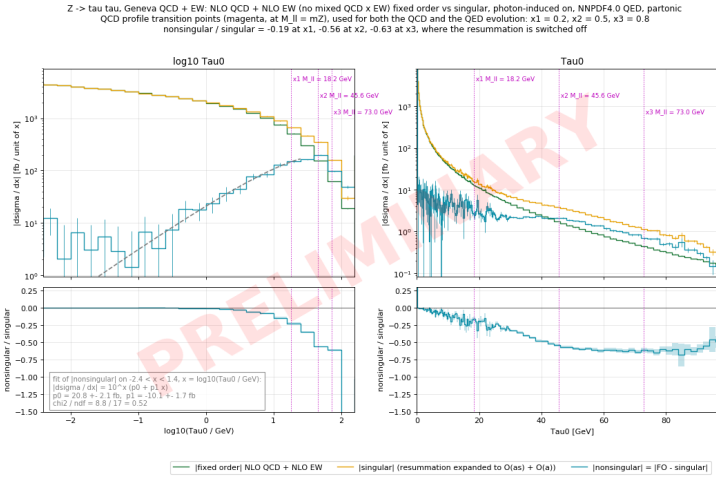
- $\mathcal{T}_0 \lesssim 50$ GeV ($x < x_1$, **canonical/resummation region**): μ_B, μ_S follow strict canonical scaling; singular dominates, nonsingular power-suppressed
- $x_1 < x < x_3$ (**transition**): $f_{\rm run}$ smoothly interpolates, and singular and nonsingular are comparably important
- $\mathcal{T}_0 \gtrsim 80$ GeV ($x > x_3$, **FO region**): $f_{\rm run} \to 1$, so $\mu_S = \mu_B = \mu_H = \mu_{\rm FO}$ and resummation is switched off restoring the correct fixed-order result

EW Resummation and profiles



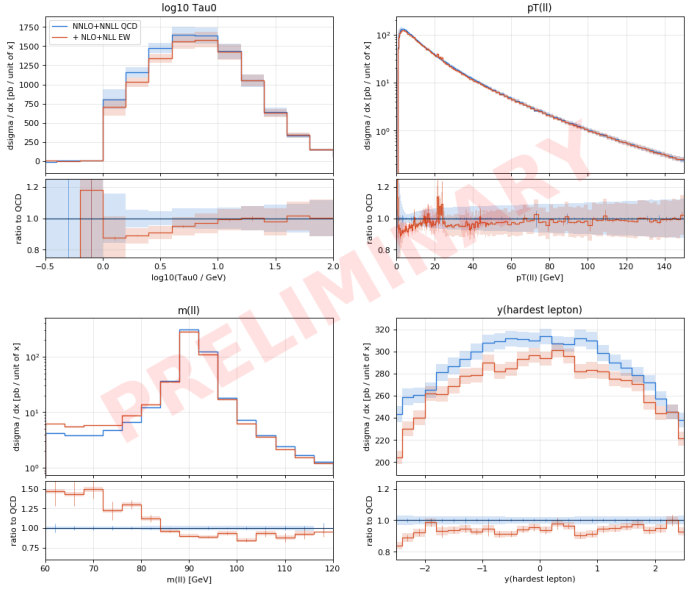
- FO tiny, singular stays $\mathcal{O}(1)$ to beyond 130 GeV, so nonsingular cancels it completely (ratio $\to -1$) already by $\sim$40 GeV, meaning EW resummation switch off much earlier than QCD

QCD+EW Resummation and profiles



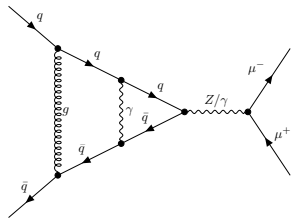
- **Joint profile** for QCD and EW resummation: QCD profile dominates over EW

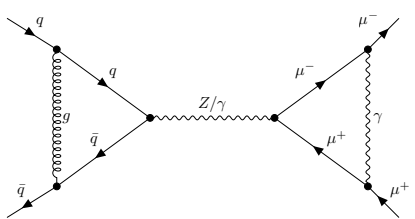
Z -> tau tau, Geneva matched NNLO+NNLL QCD vs + NLO+NLL EW, photon-induced on, NNPDF4.0 QED, partonic (no shower)
EW. NLO EW (O(alpha)) on the Drell-Yan Born only, MixedWithQCD off, no EW corrections to V + jet: no EW Sudakov logs at high pT(ll)
ATLAS fiducial: NNLO+NNLL QCD: 799.8 +- 2.2 (stat) +15.5 -15.5 (scale) pb + NLO+NLL EW: 722.5 +- 5.9 (stat) +14.0 -14.0 (scale) pb



Outlook: Mixed QCD×EW

- To upgrade GENEVA's accuracy from NLO EW to a genuine $\mathcal{O}(\alpha_s\alpha_e)$-matched NNLO QCD + NLO EW + mixed QCD×EW prediction, we need:
  - ✓ Vj + QED emissions vs Vγ + QCD emissions overlap
  - ✓ cusp/noncusp anomalous dimensions
  - ✓ coupled α_s–α_e running [1907.02971]
  - ✓ massive-lepton factorization evolution kernels
  - ✓ hard/beam/soft functions
  - ✓ Two-loop mixed QCD×EW virtuals now known for **both** neutral current [2106.11953] [2412.16095] and **charged current** [2405.00612] Drell-Yan



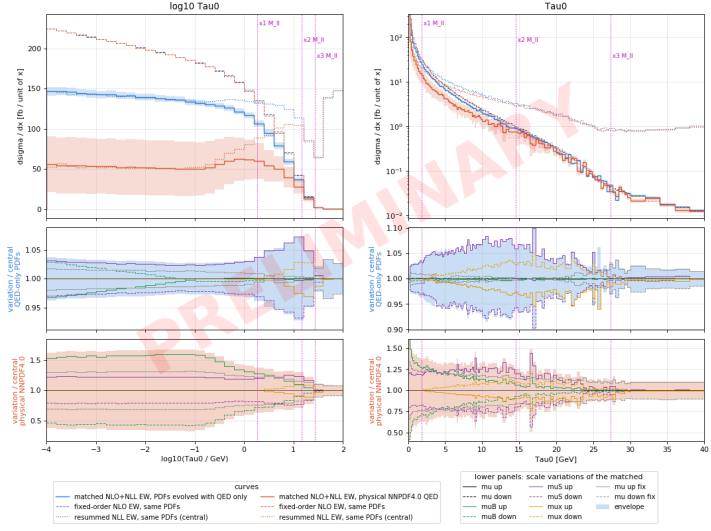


Summary

- GENEVA combines NNLO QCD, NNLL′ resummation, and a parton shower via N-jettiness subtraction, with a well-established singular/nonsingular validation methodology
- EW corrections to Drell-Yan are no longer sub-leading at the precision frontier: comparable in size to NNLO QCD uncertainties
- We are extending this to NLO EW via a *joint* SCET factorization by summing QCD and EW channels inside the same hard/beam/soft structure, using massive leptons to retain $\mathcal{T}_0$

Thank you!

Backup: Scale variation uncertainty EW only



PRELIMINARY

Backup: NNLO MC cross sections: Born, virtual, double-virtual

Fixed-order content of the N-jet GENEVA MC cross sections [1508.01475] [1909.02026]:

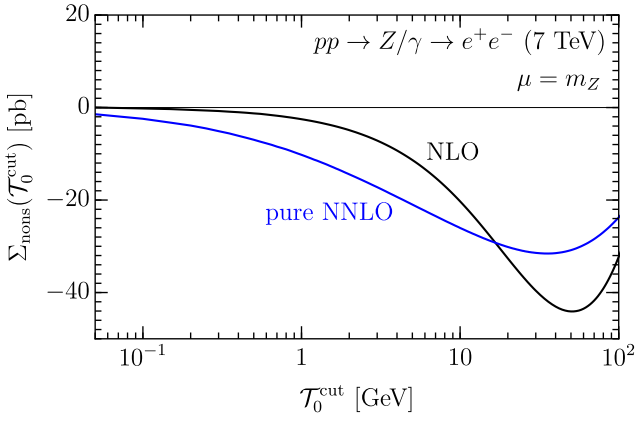
$$\frac{\mathrm{d}\sigma_0^{\mathrm{MC}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = (B_0 + V_0 + W_0)(\Phi_0) + \int \frac{\mathrm{d}\Phi_1}{\mathrm{d}\Phi_0}\,(B_1 + V_1)(\Phi_1)\,\theta[\mathcal{T}_0(\Phi_1) < \mathcal{T}_0^{\mathrm{cut}}] + \int \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_0}\,B_2(\Phi_2)\,\theta[\mathcal{T}_0(\Phi_2) < \mathcal{T}_0^{\mathrm{cut}}]$$

$$\frac{\mathrm{d}\sigma_1^{\mathrm{MC}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0{>}\mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = (B_1 + V_1)(\Phi_1)\,\theta[\mathcal{T}_0(\Phi_1) > \mathcal{T}_0^{\mathrm{cut}}] + \int \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1^{\mathcal{T}}}\,B_2(\Phi_2)\,\theta[\mathcal{T}_0(\Phi_2) > \mathcal{T}_0^{\mathrm{cut}}],$$

$$\frac{\mathrm{d}\sigma_{\geq 2}^{\mathrm{MC}}}{\mathrm{d}\Phi_2} = B_2(\Phi_2)$$

- B_N: N-parton tree-level. V_N: N-parton one-loop. W_0: **two-loop (double-virtual)**, only at Born kinematics Φ_0. This is exactly the full fixed order piece bolted onto the resummation in the matching formula, *not* part of the NNLL′ resummation's own FO-expansion/matching coefficients
- Modern equivalent language [2504.11357]: δNNLO $= W(\Phi_N) + RV(\Phi_{N+1}) + RR(\Phi_{N+2})$ (double-virtual, real-virtual, double-real)

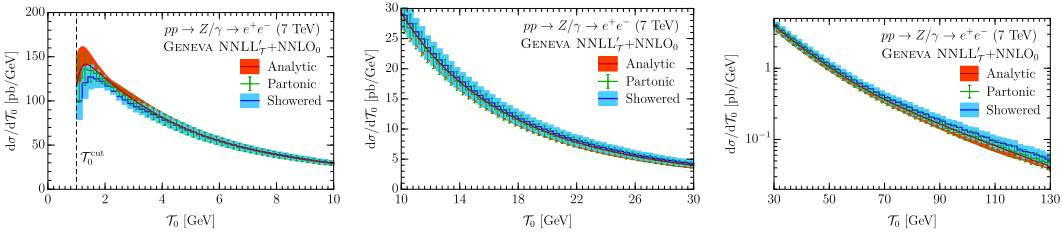
Backup: nonsingular cumulant vs. cutoff



[1508.01475]

- NLO (black) and pure NNLO (blue) nonsingular cumulant $\Sigma_{\rm nons}(\mathcal{T}_0^{\rm cut})$
- At the default $\mathcal{T}_0^{\rm cut} = 1$ GeV these terms are small but not negligible, and are included on-the-fly (NLO) or via total-rate rescaling (NNLO)

Backup: $\mathcal{T}_0$ spectrum validation chain



[1508.01475]

- Peak / transition / tail regions: analytic NNLL′+NNLO resummation vs. GENEVA partonic and showered results (before nonperturbative effects)

Backup: splitting function on-the-fly, massive emitter

$$P_{N\to N+1}(\Phi_{N+1}) = \frac{f_{kj}(\Phi_N, \mathcal{T}_N, z, \phi)}{\sum_{kj}\int_{z_{\min}(\mathcal{T}_N)}^{z_{\max}(\mathcal{T}_N)} f_{kj}(\Phi_N, \mathcal{T}_N, z, \phi)\, J(\Phi_N, \mathcal{T}_N, z)\, dz\, d\phi}\,, \qquad J = \frac{d\Phi_{N+1}}{d\Phi_N\, d\mathcal{T}_N\, dz\, d\phi}$$

- f_{kj}: on-the-fly Altarelli-Parisi kernels, recomputed per Φ_{N+1}
- Assumed there J is ϕ-independent for $1\to0, 2\to1$ mappings, which **breaks down** for a massive FSR emitter (e.g. γ off a charged lepton): $(p_f+p_\gamma)^2-m_f^2$ not fixed by $(\mathcal{T}_N, z)$ alone $\Rightarrow$ f_{kj} depends on a third, non-azimuthal coordinate ν
- Abelianzied massive quasi-collinear kernel [hep-ph/0201036]:

$$f_{kj} \;\longrightarrow\; Q_f^2\left[\frac{1+z^2}{1-z} - \frac{2m_f^2}{(p_f+p_\gamma)^2 - m_f^2}\right]$$

  second term: dead-cone piece regulating the $z \to 1$ collinear pole
- GENEVA restores a ν-independent f_{kj} via 4-point Gauss-Legendre averaging over ν's physical domain, using a $\nu \leftrightarrow 2\pi-\nu$ mirror symmetry

Backup: what SCET and `SCETlib` actually provide

- **SCET** (Soft-Collinear Effective Theory) factorizes a cross section near an IR-singular limit ($\mathcal{T}_0 \to 0$ here) into separate hard, collinear (beam), and soft functions, each depending on a single characteristic scale
- Each function obeys its own RGE: solving it sums the large logs of the scale ratios (μ_H/μ_B, μ_B/μ_S, ...) to all orders, which is exactly the resummation
- `SCETlib` is the numerical library that supplies, for a given process, *the actual ingredients* that formalism needs:
  - fixed-order matching coefficients for H, B, S at the orders GENEVA requires
  - cusp and noncusp anomalous dimensions, i.e. the RGE kernels themselves
  - numerical evolution between the canonical and common scales, including the profile-scale machinery
- GENEVA calls `SCETlib` as an external library: the factorization and matching logic live in GENEVA, the perturbative ingredients and their evolution live in `SCETlib`

Backup: GENEVA development timeline

2008 GenEvA I: original event-generation framework proposal [0801.4026]

2013 Master formula: matching fully differential NNLO to a parton shower [1311.0286]

2015 First physics application: Drell-Yan at NNLL′+NNLO matched to a shower [1508.01475]

2019 Higgsstrahlung [1909.02026]; resummed hadronic Higgs decays [2009.13533]

2020 Diphoton production [2010.10498]

2021 q_T/N^{3}LL resummation variable [2102.08390]; massive-particle ($t\bar{t}$) factorization [2111.03632], later reused for massive leptons in this talk

2022–23 Double Higgs [2212.10489]; refined gluon-fusion Higgs [2301.11875]

2024–25 Nonlocal-subtraction reformulation, explicitly flags EW as the natural next step [2504.11357]; transverse-observable resummation [2505.14773]; double Higgs with top-mass corrections [2507.08558]

2026 **This talk**: joint QCD+EW SCET factorization, massive leptons, coupled α_s–α_e running