

Photon Conversion to Low-Mass Dileptons: Comparing Kroll-Wada and Parton Shower Approaches [2506.23162]

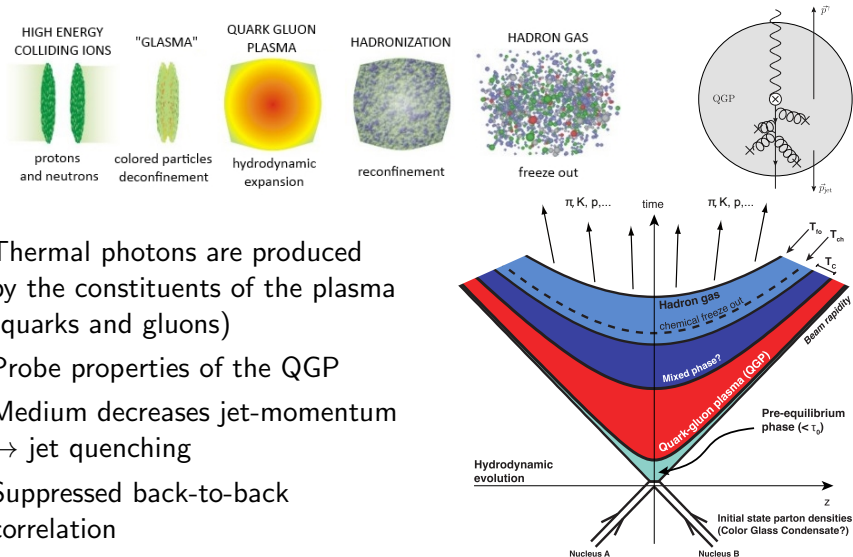
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UNIMIB & INFN

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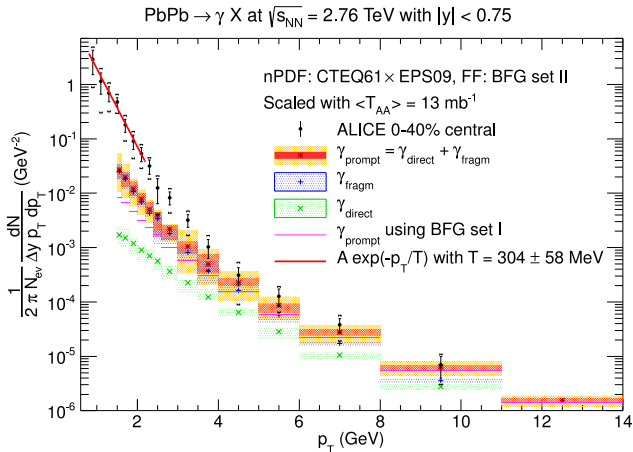
Joint INFN-UNIMI-UNIMIB Pheno Seminar

Photons as probe for Quark Gluon Plasma



- Thermal photons are produced by the constituents of the plasma (quarks and gluons)
- Probe properties of the QGP
- Medium decreases jet-momentum $\rightarrow$ jet quenching
- Suppressed back-to-back correlation

QGP temperature extraction [1307.7034]



- Good agreement in higher p_T perturbative region
- Excess photons are thermal and follow an exponential distribution
- Low p_T theory dominated by imprecise fragmentation and experimentally difficult due to photons from pion decays

Motivation for low invariant mass dileptons

- Invariant mass: Approximate time of collision stages
- No need for isolation cuts, as for real photons

HIGH ENERGY
COLLIDING IONS



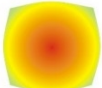
protons
and neutrons

"GLASMA"



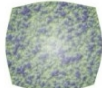
colored particles
deconfinement

QUARK GLUON
PLASMA



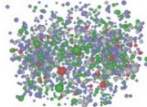
hydrodynamic
expansion

HADRONIZATION

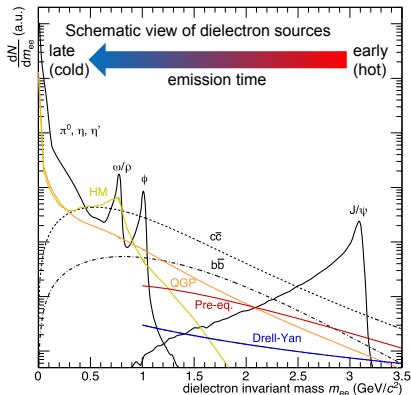
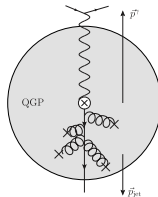


reconfinement

HADRON GAS

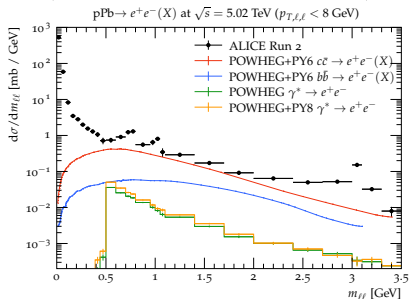
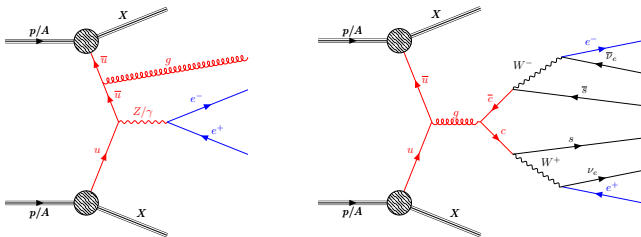


freeze out



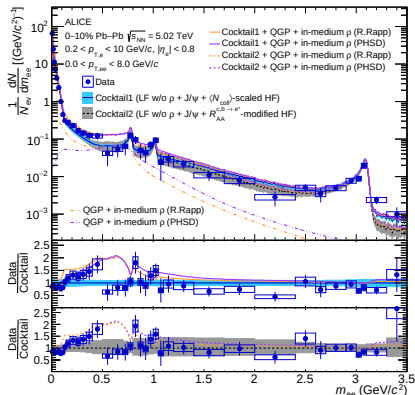
Hard dilepton production processes [2401.12875]

- Through Z/γ with a jet for non-zero $p_{T,ee}$
- Correlated open heavy-flavour $b\bar{b}$ or $c\bar{c}$ decays



Hadronic cocktail [2308.16704]

- Background cocktail consists of light-flavour (LF) decays, the J/ψ resonance, and open heavy-flavour (HF) decays.
- Normalization is usually fitted to describe the $p_{T,ee}$ and M_{ee} distributions (by factors of about 0.75 to 2.5).
- Large uncertainty for DY due to low contribution and M_{ee} cut
- DY not included by ALICE so far

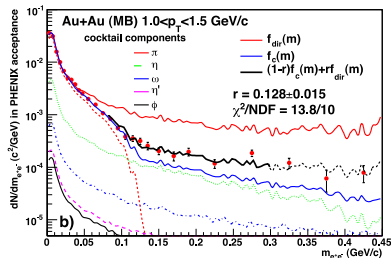
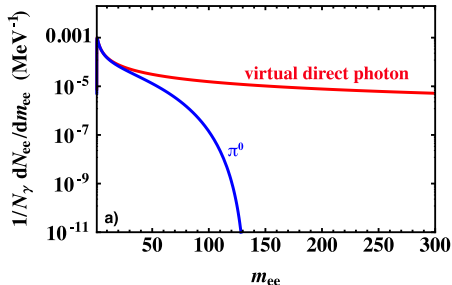


Application of Kroll-Wada in PHENIX [0908.2382]

- Virtual-photon production given by simplified Kroll-Wada equation for a given real-photon yield

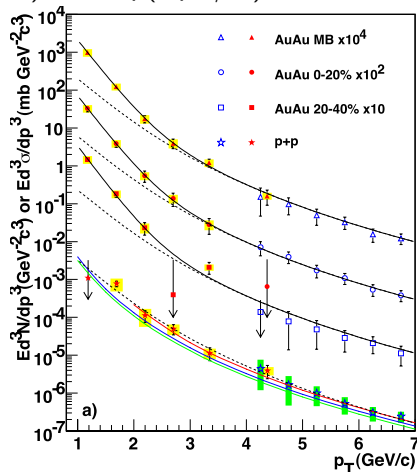
$$\frac{dN_{ee}}{dM_{ee}} = \frac{2\alpha}{3\pi} \frac{1}{M_{ee}} \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}} \left(1 + \frac{2m_e^2}{M_{ee}^2}\right) N_\gamma \approx \frac{2\alpha}{3\pi} \frac{1}{M_{ee}} N_\gamma$$

- Fit parameter $r = N_{\gamma,\text{direct}}/N_{\gamma,\text{inclusive}}$ between normalized LF cocktail (f_c) and Kroll-Wada (f_{dir})



Application of Kroll-Wada in PHENIX [0908.2382]

- Internal conversion method (Kroll-Wada equation) relates virtual photons to real photons $\Rightarrow$ measure real photon yield at lower p_T (filled points)
- pp data along $f(p_T) = A(1 + p_T^2/b)^{-n}$
- AuAu excess $f(p_T) + B \exp(-p_T/T)$, $T \approx 221$ MeV



What is the Kroll-Wada formula? [PhysRev.98.1355]

- First used to relate $\pi^0 \rightarrow \gamma ee$ to $\pi^0 \rightarrow eeee$ decay, but applicable to any photon source
- Conversion coefficient ρ from photons to dileptons:

$$\rho = \frac{W_{\text{pair}}}{W_\gamma} = \frac{\left\langle \left| \begin{array}{c} \text{Diagram 1: } \pi^0 \text{ decaying into } e^+e^- \text{ via } \gamma^* \text{ (labeled } \gamma^*) \text{ and } X \text{ (labeled } X \text{)} \end{array} \right| \right\rangle^2}{\left\langle \left| \begin{array}{c} \text{Diagram 2: } \pi^0 \text{ decaying into } \gamma \text{ (labeled } \gamma \text{)} \text{ and } X \text{ (labeled } X \text{)} \end{array} \right| \right\rangle^2}$$

$$= \frac{2\alpha}{3\pi} \int_{2m_e}^{\sqrt{s}} dM_{ee} \left(\frac{k'}{k} \right) \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}} \left(1 + \frac{2m_e^2}{M_{ee}^2} \right) \\ \times \frac{(\sqrt{s} + M_R)^2 + M_R^2 - M_{ee}^2}{(\sqrt{s} + M_R)^2 + M_R^2} \left[\frac{R_T}{M_{ee}} + \frac{2(\sqrt{s} + M_R)^2 M_{ee}}{(2\sqrt{s}M_R + s + M_{ee}^2)^2} R_L \right]$$

$$R_T = \frac{\int J_1^2(k') + J_2^2(k') d\Omega}{\int J_1^2(k) + J_2^2(k) d\Omega} \quad \text{and} \quad R_L = \frac{\int J_3^2(k') d\Omega}{\int J_1^2(k) + J_2^2(k) d\Omega}$$

What is the Kroll-Wada formula? [PhysRev.98.1355]

- At small M_{ee} , using $R_T \simeq \frac{k'}{k} \simeq 1 \gg R_L$ and massless recoiler $M_R = 0$ gives:

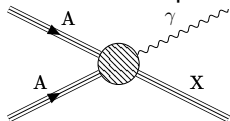
$$\begin{aligned}\frac{1}{N_\gamma} \frac{dN_{ee}}{dM_{ee}} &= \frac{2\alpha}{3\pi} \frac{1}{M_{ee}} \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}} \left(1 + \frac{2m_e^2}{M_{ee}^2}\right) \underbrace{|F(M_{ee}^2)|^2 (1 - M_{ee}^2/s)^3}_{=S \approx 1 \text{ for small } M_{ee}} \\ &\approx \frac{2\alpha}{3\pi} \frac{1}{M_{ee}}\end{aligned}$$

- $s = m_{\pi^0}^2$ and little dependence on form factor $|F_\pi(M_{ee}^2)|^2 \simeq 1$.
- Exactly true for pointlike processes (e.g., direct photons)
- Phase space important, when $M_{ee}^2 \not\ll s$.
- Alternatively, via $dM_{ee}^2/dM_{ee} = 2M_{ee}$

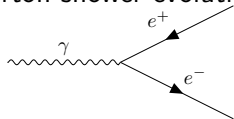
$$\frac{1}{N_\gamma} \frac{dN_{ee}}{dM_{ee}^2} \approx \frac{\alpha}{3\pi} \frac{1}{M_{ee}^2}$$

QED $1 \rightarrow 2$ parton shower splitting

Fixed order hard process:



Parton shower evolution:



- Unlike the fixed order computation the parton shower can be used for low scales ($Q \sim M_{ee}$)
- Provided production of photons/observable is IR-safe, there is no need for a normalisation
- Parton shower splitting probability:

$$\begin{aligned} d\mathcal{P}_{\gamma \rightarrow ee} &= \frac{\alpha}{2\pi} \frac{dQ^2}{Q^2} dz P_{\gamma \rightarrow ee}(z) \\ &= \frac{\alpha}{2\pi} \frac{dQ^2}{Q^2} dz e_e^2 \left[z^2 + (1-z)^2 + 2 \frac{m_e^2}{M_{ee}^2} \right] \end{aligned}$$

- Phase space limits:

$$z_{\pm} = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}}$$

Evolution variable [2210.07007]

- Splitting variable y , light-cone momentum fraction z and dipole invariant mass $s = (p_\gamma + p_R)^2 = (p_{ee} + p_R)^2$
- Transverse momentum k_T^2 ordering

$$Q^2 = k_{T,\gamma \rightarrow ee}^2 = (s - 2m_e^2)yz(1 - z) - m_e^2(z^2 + (1 - z)^2)$$

- Photon splitting threshold at $k_T^2 = 0 \implies$ infrared cutoff $Q_0 \neq 0$ would induce a minimal k_T and opening angle
- Virtuality M_{ee}^2 ordering

$$Q^2 = M_{ee}^2 = (s - 2m_e^2)y + 2m_e^2$$

- Splitting probability with $Q = M_{ee}$:

$$\frac{d\mathcal{P}_{\gamma \rightarrow ee}}{dM_{ee}^2} = \frac{\alpha e_e^2}{3\pi} \frac{1}{M_{ee}^2} \left(1 + 2 \frac{m_e^2}{M_{ee}^2} \right) \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}}$$

Vincia 2 $\rightarrow$ 3 parton shower splitting

- Antenna function for $\gamma k \rightarrow (ij)k$ splitting ($ij = ee$, $k = \text{recoiler}$):

$$P_{\gamma k \rightarrow ijk} = \frac{1}{s} \frac{1}{2} \frac{1}{y_{ij} + 2\mu_e^2} \left[y_{ik}^2 + y_{jk}^2 + \frac{2\mu_e^2}{y_{ij} + 2\mu_e^2} \right]$$

with $y_{ik} = \frac{s_{ik}}{s} \simeq z$, $y_{jk} \simeq 1 - z$ in the collinear limit, and $\mu_e^2 = \frac{m_e^2}{s}$.

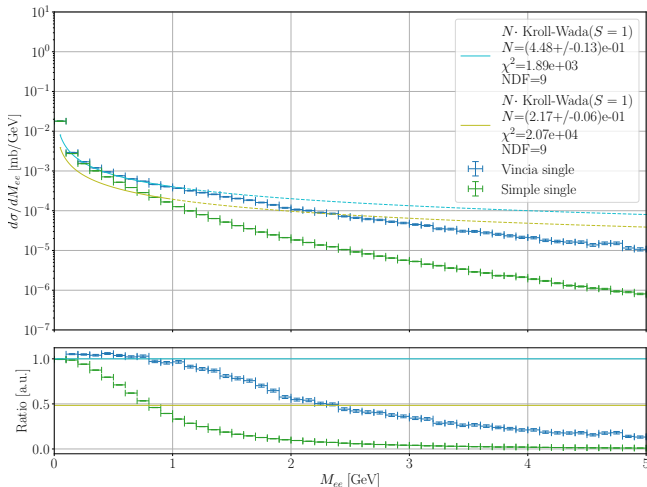
- Integrated probability:

$$\begin{aligned} \frac{d\mathcal{P}_{\gamma \rightarrow ee}}{dM_{ee}^2} &= \sqrt{\frac{M_{ee}^6 - 2M_{ee}^4 m_e^2 - 2M_{ee}^4 s + 8M_{ee}^2 m_e^2 s + M_{ee}^2 s^2 - 6m_e^2 s^2}{M_{ee}^2 - 2m_e^2}} \\ &\times \frac{\alpha e_e^2 (-M_{ee}^8 + 2M_{ee}^6 m_e^2 + 2M_{ee}^6 s - 5M_{ee}^4 m_e^2 s - M_{ee}^4 s^2 + 6m_e^4 s^2)}{3\pi M_{ee}^4 s^3 (-M_{ee}^2 + 2m_e^2)} \end{aligned}$$

- Phase space factor visible in massless electron limit:

$$\frac{d\mathcal{P}_{\gamma \rightarrow ee}}{dM_{ee}^2} = \frac{\alpha e_e^2}{3\pi} \frac{1}{M_{ee}^2} \left(1 - \frac{M_{ee}^2}{s} \right)^3$$

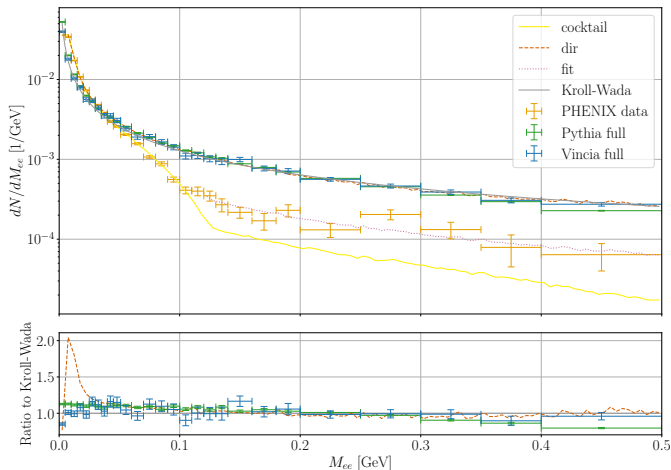
Analytic Kroll Wada vs Monte Carlo Pythia/Vincia



- Vincia antenna shower agrees with Kroll-Wada formula below 1 GeV for $\sqrt{S_{pp}} = 5.02$ TeV
- Phase space factor $(1 - M_{ee}^2/s)^3$ drives suppression above 1 GeV

Dielectron PHENIX yields [0912.0244]

- Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- Fitted $r = N_{\gamma,\text{direct}}/N_{\gamma,\text{inclusive}} = 0.19$
- Our predictions do not include PHENIX acceptance and detector smearing effects

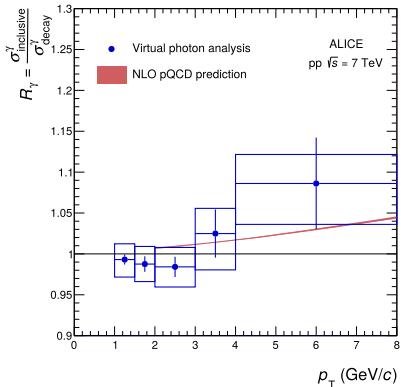
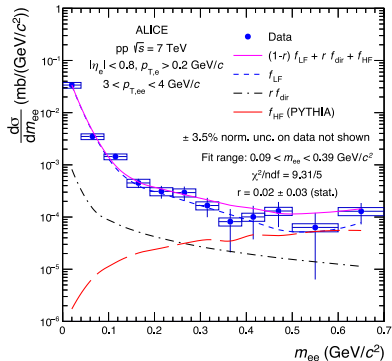


Application of Kroll-Wada in ALICE [1805.04391]

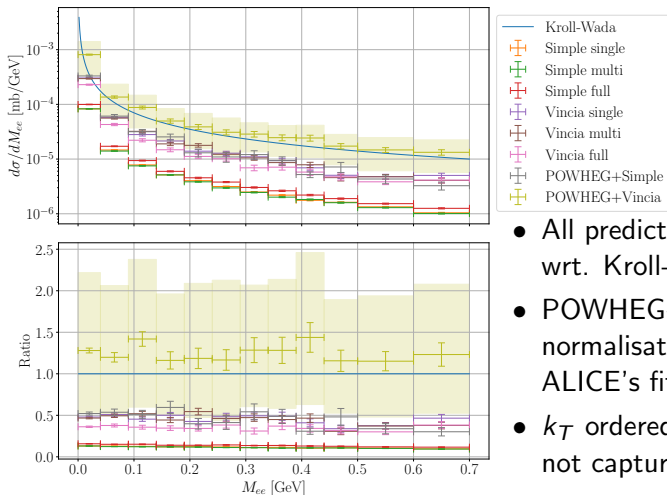
- Cross sections instead of yields
- Additional heavy flavour contribution
- Extracted $R_\gamma = 1/(1 - r) = \sigma_{\text{incl}}/\sigma_{\text{decay}}$ can be compared against pQCD

$$r = \frac{d\sigma_\gamma^{\text{direct,NLO}}(p_T)}{d\sigma_\gamma^{\text{incl}}(p_T)} \quad \text{using}$$

$$\frac{dN_\gamma^{\text{incl}}}{dp_T} = \frac{N_{ee}^{\text{data}}}{N_{ee}^{\text{cocktail}}} \times \frac{dN_\gamma^{\text{cocktail}}}{dp_T}$$



Dielectron ALICE cross section [1805.04391]

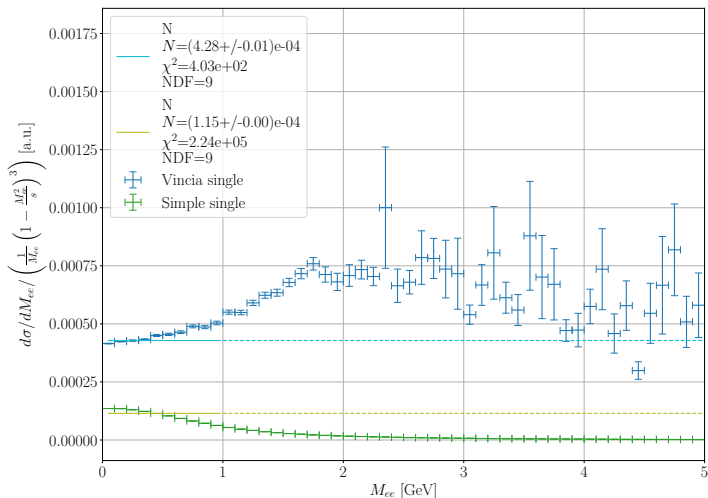


- All predictions are relatively flat wrt. Kroll-Wada($S=1$)
- POWHEG+VINCIA normalisation agrees with ALICE's fitted Kroll-Wada
- k_T ordered simple shower does not capture normalisation
- Large scale uncertainties from direct photon production in POWHEG due to low p_T
- Large K-factor from $LO \rightarrow NLQ$

Conclusion

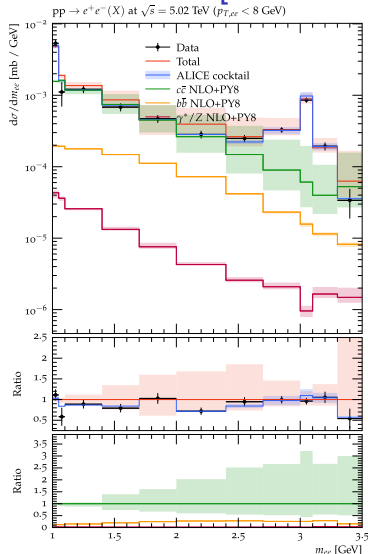
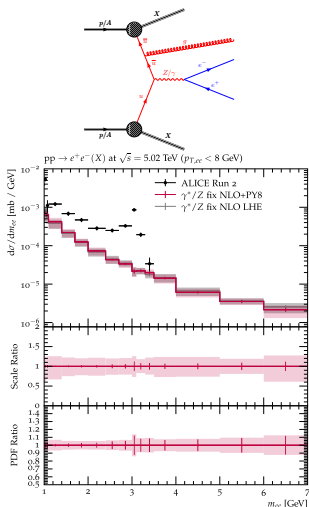
- Modern splitting functions and parton showers reproduce Kroll-Wada in the LMR, with explicit treatment of subleading form factor $S \implies$ Analytic Kroll-Wada formula can be used to validate $\gamma \rightarrow ee$ parton shower implementation
- Phase space factor $(1 - M_{ee}^2/s)^3$ important even at collider energies
- The approach accounts for higher-order effects, logarithmic corrections, phase-space cuts, acceptance, and detector effects
- POWHEG+VINCIA provides reliable NLO normalisation instead of need for fitting r in Kroll-Wada
- POWHEG+SIMPLE doesn't (can't) have the correct normalisation due to k_T ordering instead of M_{ee} virtuality ordering
- Future: Matching with virtual photon production in IMR and HMR [2401.12875] with POWHEG+Vincia in the LMR [2506.23162]

Backup: Analytic vs Monte Carlo



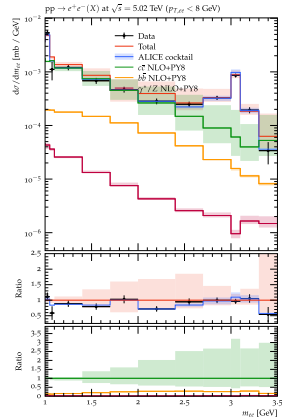
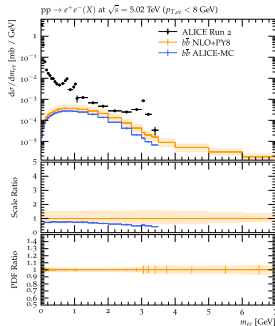
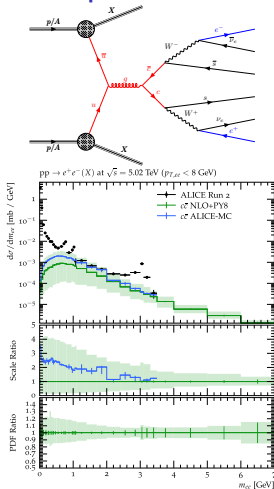
- Vincia behaviour follows phase space factor, with a smaller χ^2/NDF than previous Kroll-Wada(S=1)

Backup: DY contribution to IMR and HMR [2401.12875]



- Combined total fit (w/o LMR) gives scaling by 0.23(17) (central) and 0.06(20) (shape) for DY

Backup: Hadronic cocktail uncertainties [2401.12875]



- Large scale uncertainties for dominant $c\bar{c}$ contribution
- Shape uncertainty, each contribution normalized individually, shifts uncertainty away from $M_{ee} \approx m_c$ towards larger masses