

From Kroll-Wada to Parton Showers: Modern Approaches to Photon-to-Dilepton Conversion for ALICE [2506.23162]

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What is the Kroll-Wada formula?

- Conversion coefficient ρ from photons to dileptons:

$$\rho = \frac{W_{\text{pair}}}{W_{\gamma}} = \frac{\text{Diagram 1}}{\text{Diagram 2}}$$

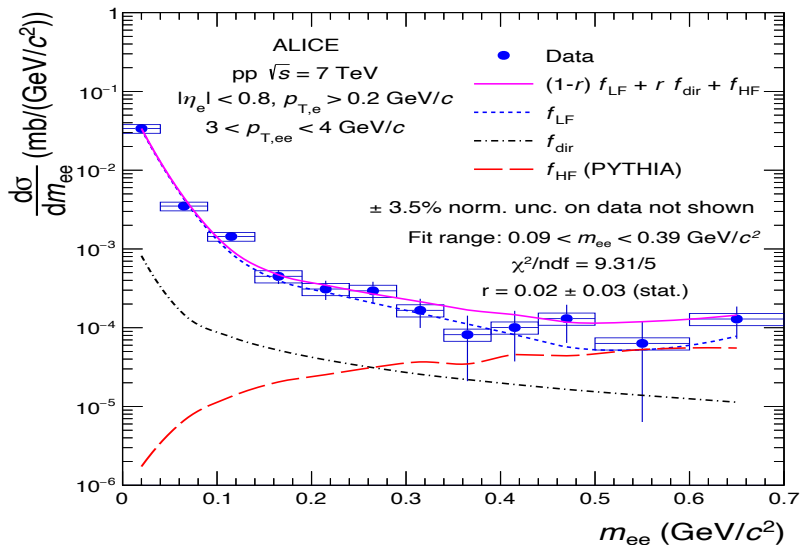
The top diagram shows a virtual photon (γ^*) decaying into an electron-positron pair ($e^+ e^-$). The bottom diagram shows a real photon (γ) decaying into a dilepton pair (X). Both diagrams have incoming particles labeled A and outgoing particles labeled X.

- Used to relate number of real photons N_{γ} to number of dileptons N_{ee} :

$$\frac{1}{N_{\gamma^*}} \frac{dN_{ee}}{dM_{ee}} = \frac{2\alpha}{3\pi} \frac{1}{M_{ee}} \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}} \left(1 + \frac{2m_e^2}{M_{ee}^2}\right) \underbrace{|F(M_{ee}^2)|^2 (1 - M_{ee}^2/s)^3}_{=S \approx 1 \text{ for small } M_{ee}}$$

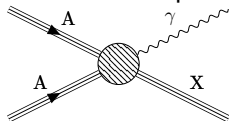
$$\approx \frac{2\alpha}{3\pi} \frac{1}{M_{ee}}$$

Application of Kroll-Wada in ALICE [1805.04391]

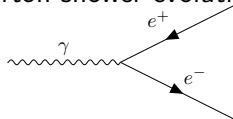


QED $1 \rightarrow 2$ parton shower splitting

Fixed order hard process:



Parton shower evolution:



- Unlike the fixed order computation the parton shower can be used for low scales ($Q = M_{ee}$)
- Provided production of photons/observable is IR-safe, there is no need for a normalisation
- Parton shower splitting probability:

$$\begin{aligned} d\mathcal{P}_{\gamma \rightarrow ee} &= \frac{\alpha}{2\pi} \frac{dQ^2}{Q^2} dz P_{\gamma \rightarrow ee}(z) \\ &= \frac{\alpha}{2\pi} \frac{dQ^2}{Q^2} dz e_e^2 \left[z^2 + (1-z)^2 + 2 \frac{m_e^2}{Q^2} \right] \end{aligned}$$

- Evolution variable $Q = M_{ee}$ (virtuality ordering):

$$\frac{d\mathcal{P}_{\gamma \rightarrow ee}}{dM_{ee}^2} = \frac{\alpha e_e^2}{3\pi} \frac{1}{M_{ee}^2} \left(1 + 2 \frac{m_e^2}{M_{ee}^2} \right) \sqrt{1 - \frac{4m_e^2}{M_{ee}^2}}$$

Vincia 2 $\rightarrow$ 3 parton shower splitting

- Antenna function for $\gamma k \rightarrow (ij)k$ splitting ($ij = ee$, $k = \text{recoiler}$):

$$P_{\gamma k \rightarrow ijk} = \frac{1}{s} \frac{1}{2} \frac{1}{y_{ij} + 2\mu_e^2} \left[y_{ik}^2 + y_{jk}^2 + \frac{2\mu_e^2}{y_{ij} + 2\mu_e^2} \right]$$

with $y_{ik} = \frac{s_{ik}}{s} \simeq z$, $y_{jk} \simeq 1 - z$ in the collinear limit, and $\mu_e^2 = \frac{m_e^2}{s}$.

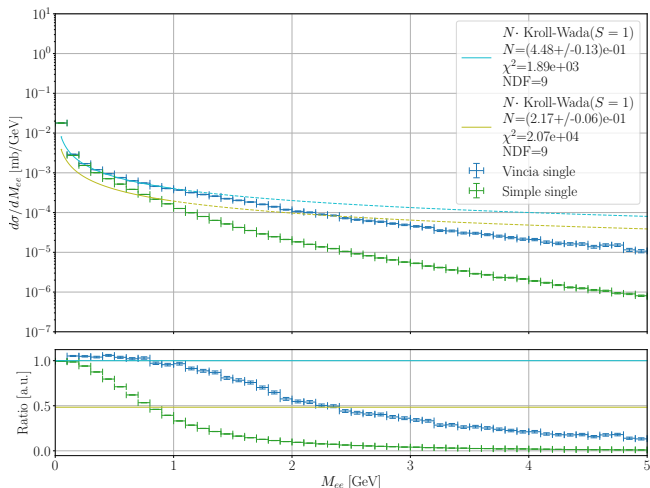
- Integrated probability:

$$\frac{d\mathcal{P}_{\gamma \rightarrow ee}}{dM_{ee}^2} = \sqrt{\frac{M_{ee}^6 - 2M_{ee}^4 m_e^2 - 2M_{ee}^4 s + 8M_{ee}^2 m_e^2 s + M_{ee}^2 s^2 - 6m_e^2 s^2}{M_{ee}^2 - 2m_e^2}}$$
$$\cdot \frac{\alpha e_e^2 (-M_{ee}^8 + 2M_{ee}^6 m_e^2 + 2M_{ee}^6 s - 5M_{ee}^4 m_e^2 s - M_{ee}^4 s^2 + 6m_e^4 s^2)}{3\pi M_{ee}^4 s^3 (-M_{ee}^2 + 2m_e^2)}$$

- Phase space factor visible in massless electron limit:

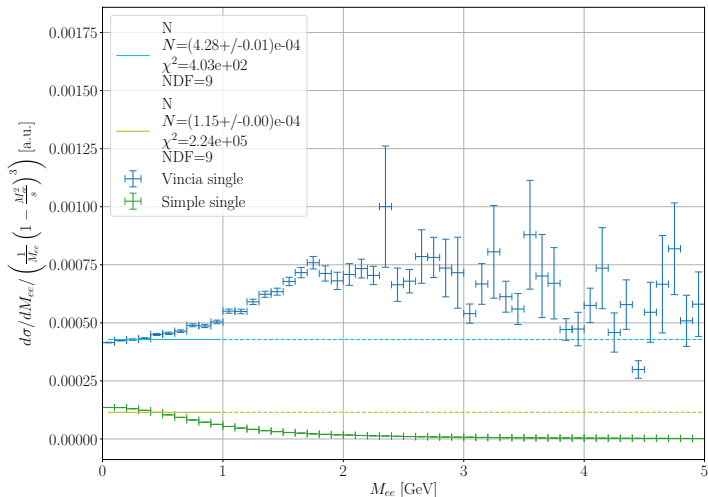
$$\frac{d\mathcal{P}_{\gamma \rightarrow ee}}{dM_{ee}^2} = \frac{\alpha e_e^2}{3\pi} \frac{1}{M_{ee}^2} \left(1 - \frac{M_{ee}^2}{s} \right)^3$$

Analytic vs Monte Carlo



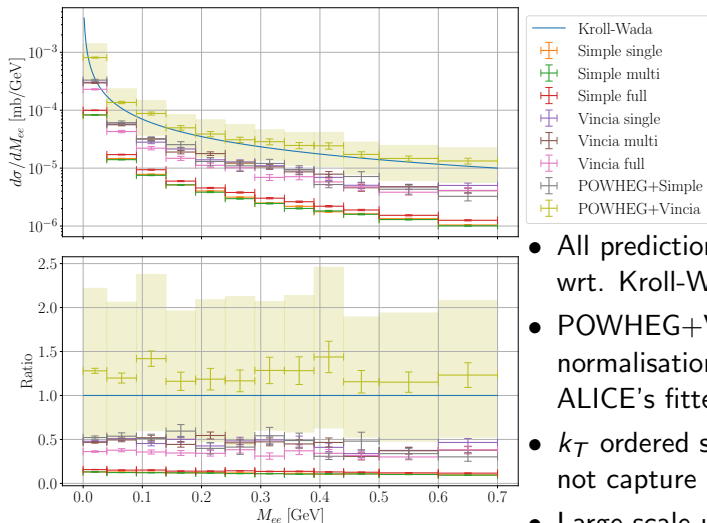
- Vincia antenna shower agrees with Kroll-Wada formula below 1 GeV for $\sqrt{S_{pp}} = 5.02$ TeV
- Phase space factor $(1 - M_{ee}^2/s)^3$ drives suppression above 1 GeV

Analytic vs Monte Carlo



- Vincia behaviour follows phase space factor, with a smaller χ^2/NDF than previous Kroll-Wada(S=1)

Dielectron ALICE predictions

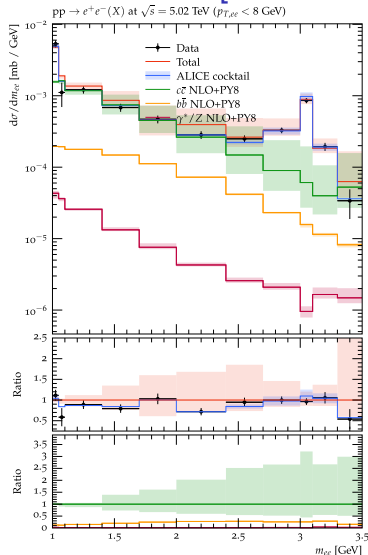
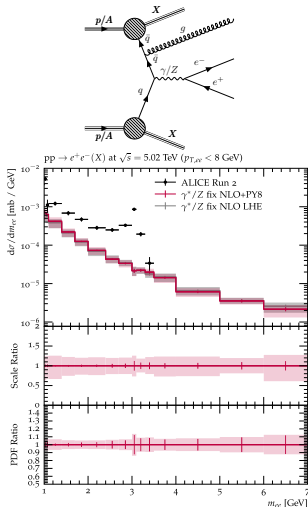


- All predictions are relatively flat wrt. Kroll-Wada($S=1$)
- POWHEG+VINCIA normalisation agrees with ALICE's fitted Kroll-Wada
- k_T ordered simple shower does not capture normalisation
- Large scale uncertainties from hard photon production in POWHEG due to low p_T
- Large K-factor from $\text{LO} \rightarrow \text{NLO}$

Conclusion

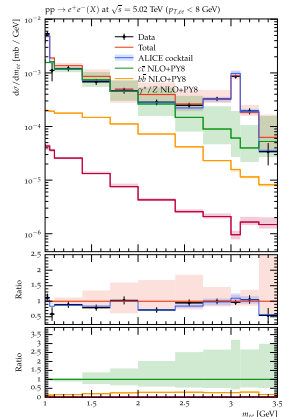
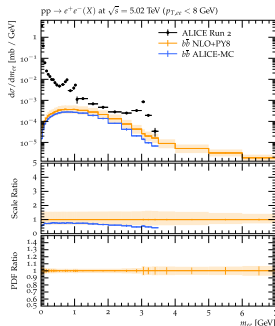
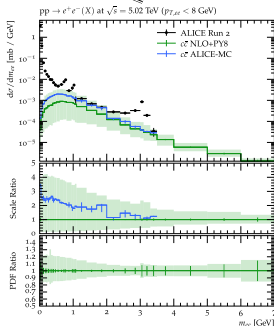
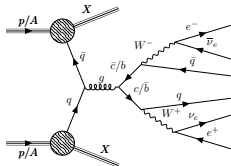
- Modern splitting functions and parton showers reproduce Kroll-Wada in the LMR, with explicit treatment of subleading form factor S
- Kroll-Wada formula is valid for pointlike processes, $S = 1$
- Phase space factor $(1 - M_{ee}^2/s)^3$ important even at collider energies
- The approach accounts for higher-order effects, logarithmic corrections, phase-space cuts, acceptance, and detector effects
- POWHEG+VINCIA provides reliable NLO normalisation instead of need for fitting in Kroll-Wada
- POWHEG+SIMPLE doesn't (can't) have the correct normalisation due to k_T ordering instead of M_{ee} virtuality ordering
- Future: Matching with virtual photon production in IMR and HMR [2401.12875] with POWHEG+Vincia in the LMR [2506.23162]

Backup: DY contribution to IMR and HMR [2401.12875]



- Combined total fit (w/o LMR) gives scaling by 0.23(17) (central) and 0.06(20) (shape) for DY

Backup: Hadronic cocktail uncertainties [2401.12875]



- Large scale uncertainties for dominant $c\bar{c}$ contribution
- Shape uncertainty, each contribution normalized individually, shifts uncertainty away from $M_{ee} \approx m_c$ towards larger masses